AC 2012-3699: ENCOURAGING DIVERGENT THINKING

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Daniel Raviv is a professor of computer and electrical engineering and computer science at Florida Atlantic University. He also served as Assistant Provost for Innovation and Entrepreneurship. With more than 25 years of combined experience in the high-tech industry, government, and academia, Raviv developed fundamentally different approaches to ”out-of-the-box” thinking and a breakthrough methodology known as ”Eight Keys to Innovation.” He has been sharing his contributions with professionals in businesses, academia, and institutes nationally and internationally. Most recently, he was a visiting professor at the University of Maryland (at Mtech, Maryland Technology Enterprise Institute) and at Johns Hopkins University (at the Center for Leadership Education), where he researched and delivered processes for creative and innovative problem solving. For his unique contributions, he received the prestigious Distinguished Teacher of the Year Award, the Faculty Talon Award, the University Researcher of the Year AEA Abacus Award, and the President's Leadership Award. Raviv has published in the areas of vision-based driverless cars, green innovation, and innovative thinking. He is a Co-holder of a Guinness World Record. Raviv received his Ph.D. degree from Case Western Reserve University in 1987 and M.Sc. and B.Sc. degrees from the Technion - Israel Institute of Technology in 1982 and 1980, respectively.

Encouraging Divergent Thinking

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Abstract

An important aspect of innovative problem solving is ideation. Ideation renders diverse ideas to emerge, a combination of which can be used to solve a given problem. It allows students to explore multiple solutions, and more importantly to realize that usually there is no "one correct answer" to a given problem.

This paper focuses on team-based, interpersonal, individual hands-on activities aimed at encouraging divergent thinking. The activities allow students to change their point of view, avoid unnecessary assumptions, think outrageously and unexpectedly, and improvise using limited available resources. Students are encouraged to find multiple, imaginative, intuitive, as well as common-sense solutions.

The activities are designed to accommodate multiple teaching/learning scenarios, such as individual settings where each and every student is challenged with a specific problem; team settings that promote group divergent thinking, discussions and competitions; and, collectively, where all students generate ideas for a given challenge.

Some activities are designed to be self-paced; others have strict time constraints, leading to ideation under pressure. The instructions for the activities are very clear and concise allowing participants to be relieved from unnecessary constraints or assumptions. Following each activity, a short discussion session is facilitated to reflect on the activity's goals, challenges and results. Even though some of the activities may not be new, they are still introduced with different twists and/or with a new set of instructions.

This paper describes a collection of different activities and their purpose, as well as multiple solutions as shared by participants, some of which are unexpected, explorative and unconventional. Benefits acquired by the students are identified. An example for an activity is: *Make the following sentence complete and correct: "This sentence has _____ letters."* By not making unnecessary assumptions, and allowing imaginations to run wild, students have come up with more than 200 different solutions!

The activities have been embedded in undergraduate and graduate courses, e.g., "Creativity and Innovation," that were taught at three different universities with participants from different majors/colleges. A goal of all the related courses is to enhance innovative and creative thinking abilities of students, resulting in skills that can be used in problem solving.

Assessment of some of the activities is detailed following the description of activities. The analyzed results are based on the average number of solutions per student, the standard deviation, and the total number of different solutions. The results clearly indicate a consistent

and significant improvement in idea generation. They show an average boost in the number of ideas by a factor of nearly two and a half produced by about 130 participants.

The author shares these activities to allow people who teach creative and innovative thinking to have “ready to use” class material, avoiding re-inventing and searching for new activities. This paper lists the “better” activities that “survived” many years of experimentation with continuous improvements.

Introduction

This paper focuses on team-based, interpersonal, individual hands-on activities aimed at encouraging divergent thinking, aka ideation. The activities encourage diverse ideas to emerge in different class settings (individual, team, communication-based, competition-based, etc.) allowing for different teaching/learning styles, and encouraging individuals to think and express themselves in self-paced or under strict time constraints. The activities allow students to change their point of view, avoid unnecessary assumptions, think outrageously and unexpectedly, and improvise using limited available resources. Students are encouraged to find multiple, imaginative, intuitive, as well as common-sense solutions. Ideation allows students not only to explore multiple solutions, but more importantly to realize that usually there is no “one correct answer” to a given problem.

Following each activity, a short class discussion session is facilitated to reflect on the activity’s goals, challenges and results. Even though some of the activities may not be new, they are still introduced with a different twist and/or with a new set of instructions. It should be noted that students’ ideas are not judged, ranked or criticized.

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The author shares these activities to allow people who teach creative and innovative thinking to have “ready to use” class material, avoiding re-inventing and searching for new activities. This paper lists the “better” activities that “survived” many years of experimentation with continuous improvements.

The activities have been used in undergraduate, graduate and high schools classes. Some of the problems/activities, specifically, those under section “I” in this paper, i.e., “Where are you?” and “The Jumping Problem” have been used to assess an aspect of students ability to generate ideas. The activities have been tested and assessed in many different classes over more than 15 years:

- Freshman and sophomore level students (all disciplines):

  “Introduction to Creativity”

  “Creativity”
- Junior and Senior level students (at home institution and other private and public universities (Johns Hopkins and University of Maryland):

"Creativity and Innovation"

- Graduate level students (mostly to engineering students):

"Innovative Thinking"

- Undergraduate and select group of high school students (as dual enrollment classes):

"Inventive Problem Solving in Engineering"

"Discoveries in Engineering: Innovative Problem Solving"

Assessment of some of the activities is detailed following the description of activities. The analyzed results are based on the average number of solutions per student, the standard deviation, and the total number of different solutions. The results clearly indicate a consistent and significant improvement in idea generation. They show an average increase in the number of ideas by a factor of nearly two and a half produced by about 130 participants.

Please refer to all ASEE papers in the Reference section.

The Activities

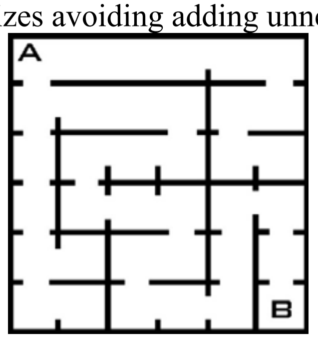
The ideation activities are grouped according to their objectives:

A) Pattern breaking
B) Inquiry-based
C) Self-paced ideation – allowing imagination to run wild
D) Ideation under limited time constraints
E) Imaginative observation
F) Visualization
G) Collective group ideation
H) Exploring problems with infinite number of solutions
I) Evaluation problems
J) Twists to well-known out-of-the-box problems
K) Exploring simple problems with unexpected solutions

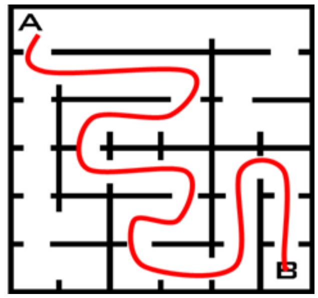
A) Pattern breaking

Trace a Path from Point A to Point B

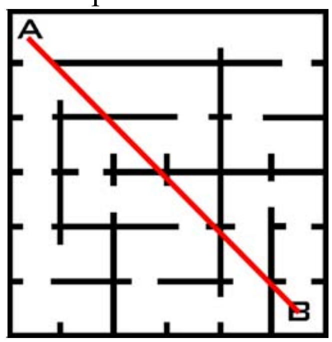
This activity emphasizes avoiding adding unnecessary assumptions



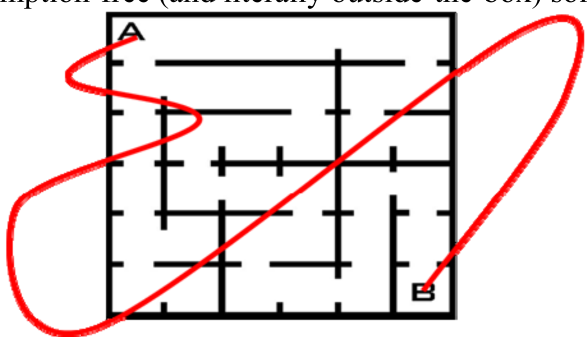
Common "expected" solution



Assumption-free solution



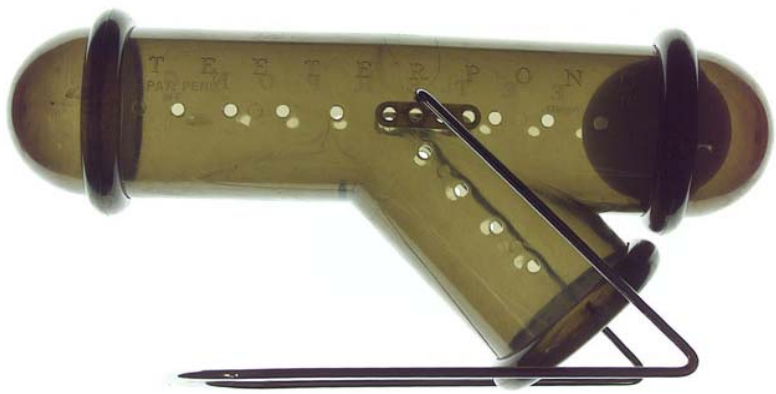
Assumption-free (and literally outside-the-box) solution



B) Inquiry-based

What is it?

Students are shown an invention, and asked to “figure out” what it is. For example:



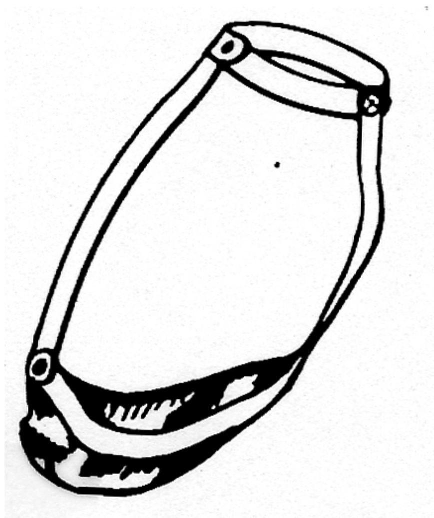
After a few minutes of guessing and discussing (usually with some hints) they discover that it is a mousetrap.

The following is the patent abstract of “Mousetrap for catching mice live.”

A "Y" shaped mousetrap lures a mouse into an open end of the "Y" by means of smelly bait located at a closed end of the bottom of the "Y". The "Y" is pivotally supported horizontally by a stand. As the mouse walks past the pivot point, a ping pong ball rolls from the opposite short "Y" tube member and down to the entrance of the open ended tube member. The mouse is trapped alive and can be drowned by immersing the mousetrap.

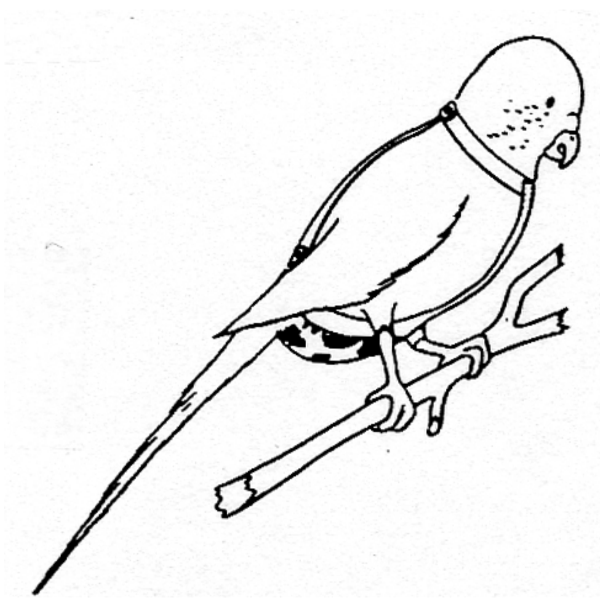
What is it?

The following is shown to students, with a brief introduction that this is a 1956 invention made out of cloth or plastic. The question is “what is it?”



In this exercise the students start with group ideation, later they are provided with a hint and/or a solution.

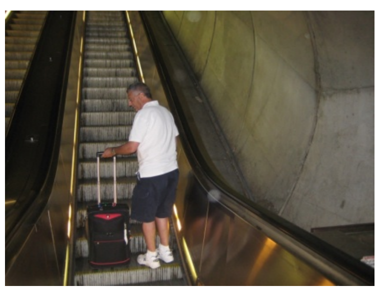
Solution:
Bird Diaper



List ideas to “how come I was able to stand like this?”

Surprisingly it is not obvious.

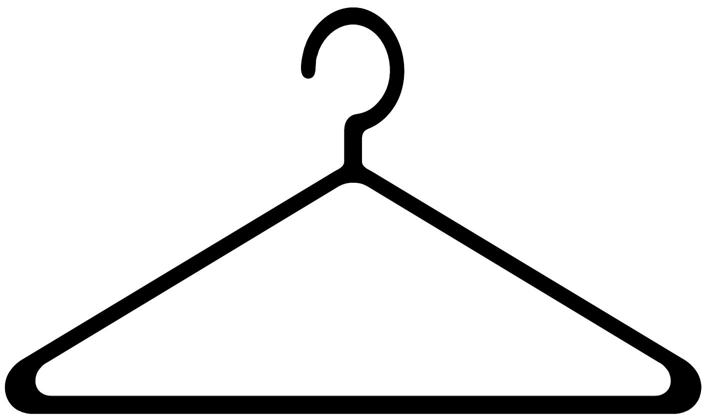
Solution: a visual presentation show the solution



C) Self-paced ideation – allowing imagination to run wild

What can be done with coat hanger?

Students are shown a coat hanger and being asked to individually list different possible uses. They are given the freedom to use any material, size or shape of a hanger; they may imagine cutting it, shrinking it, using many of them, etc. Amazingly, in a short period of time each student writes many ideas. The students take turns to mention their ideas. Usually one idea mentioned by each student is suitable time-wise and fun-wise to complete the exercise. (The coat hanger may be substituted with any other basic familiar object such as a book, or a mailbox.)



Hundreds of different ideas have been generated by students in a short period of time. For example, they suggest antenna, cup holder, box, toilet paper holder, artistic 3d figure, keychain.

Here is a visual example (sent to me in an e-mail):

What can you do with a shopping cart?

After the exercise, the following is shown, just for fun
(sent to me in an e-mail)

D) Ideation under limited time constraints

A sixty second individual exercise:
What can you do with a spoon?

The ideas are shared later in class.

A group exercise:
Describe this pencil

The pencil is moved around in class from one student to another. Each student says something about the pencil. 40 ideas are easily obtained (sometimes the instructor needs to have second and third rounds of moving the pencil around)

E) Imaginative observation

How people say “no”

On a separate piece of paper, without writing their names, students are asked to write down as many possible ways for “how people say ‘no’”.

Here are some examples of what they write:
-We would love to do it, but…
-You know, something came up, …
-We are going to do it, aren’t we?
-We could, but, …
-May be another time
-Whatever
-I’ll call you about it

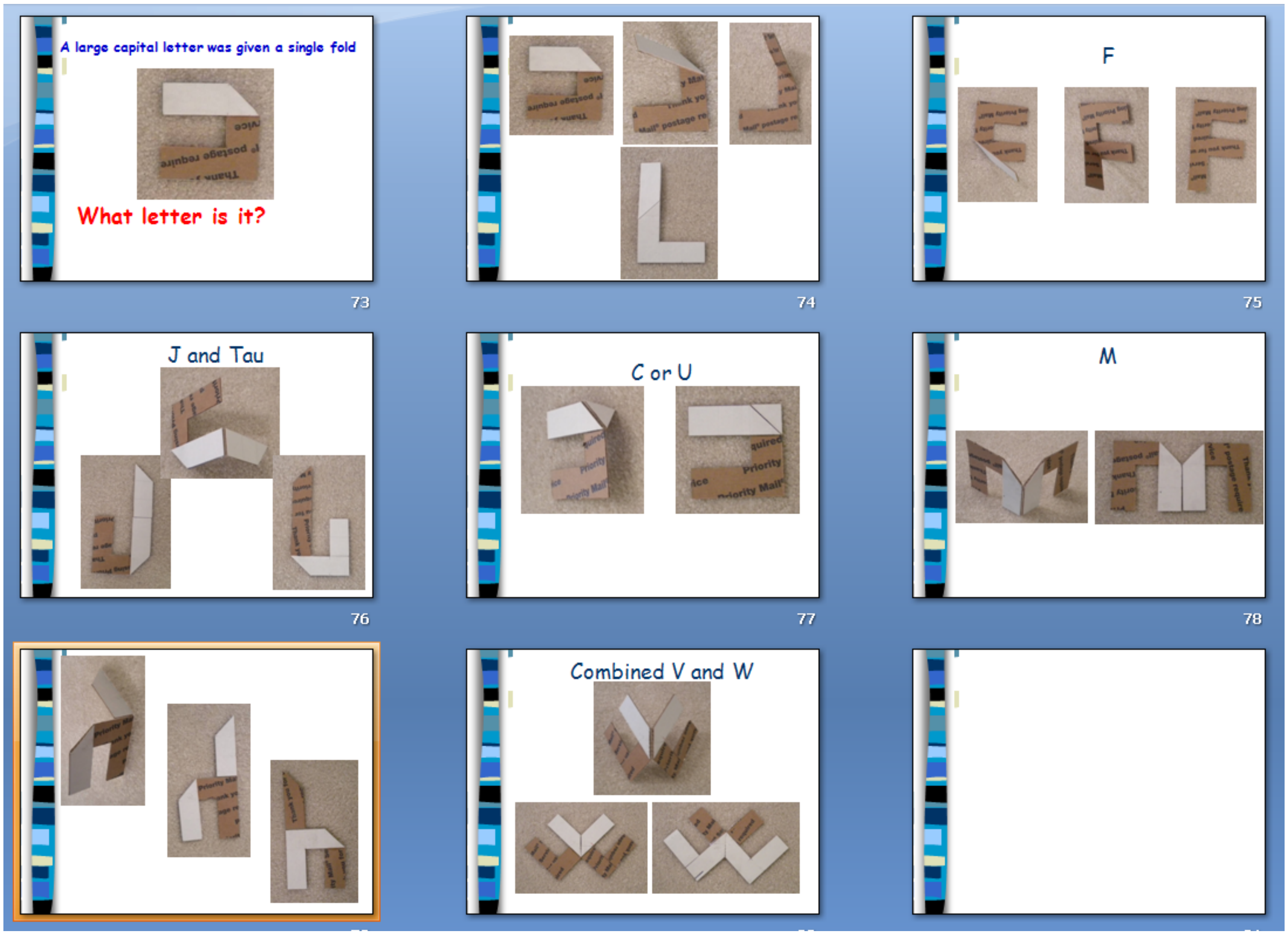
The actual lists made by the students are surprisingly long. Collectively they listed more than 200 different ways in a short period of time. This activity doesn’t only show the “no unique solution” concept but adds to the fun and enjoyable element of the class.

F) Visualization

A large capital letter was given a single fold. What letter is it?

(Note: It is useful to have the object and some solutions made out of cardboard. Visualization makes a big difference.)

Here are some solutions:



G) Collective group ideation

Problems with little or no data/information.
These kind of problems help introduce the "no right answer" to a problem.

The 120 problem

The following is a problem that works in individual and small team settings:
Use the following numbers 2, 3, 5, 10, 24 and operations such as ( ), *, :, +, -, exp(.) to get to a total of 120. Each number must be used once and only once. Operations may be used once, more than once or not at all. In addition, participants may invent their own operations (even strange ones).

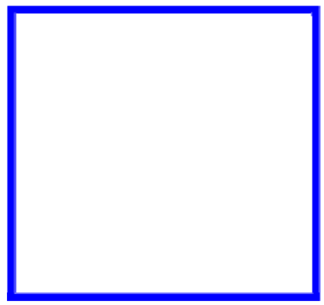
Here are some solutions:
5!+((24/3+2)-10)
((2x10) – (5x3))x120
(3-2)x24x10-5!
24x3 + 5x10 -2
5! / (24-(3+(10x2)))
24x5 / (10-(3^2))
24x10 / ((3!-5) x2)
24/2 x10 x (3!-5)
(10 x 24) – {sqrt [(2+3) x 5]} !
(5 x 24) – (10 - 3**2)
(24 x 3) + (5x10) – 2

Some unconventional solutions:
(recall that students were allowed to "invent" their own operations)
(10 x 3**2) + 24 + 5⅄
The ⅄ operation means "ceil to next integer"
In this case 5⅄ became 6
2x3x5x10x24 Ώ
Here the "Ώ" operation means "subtract 7080"
103+24-2-5
Here an operation was to attach numbers, i.e., 10 and 3 became 103

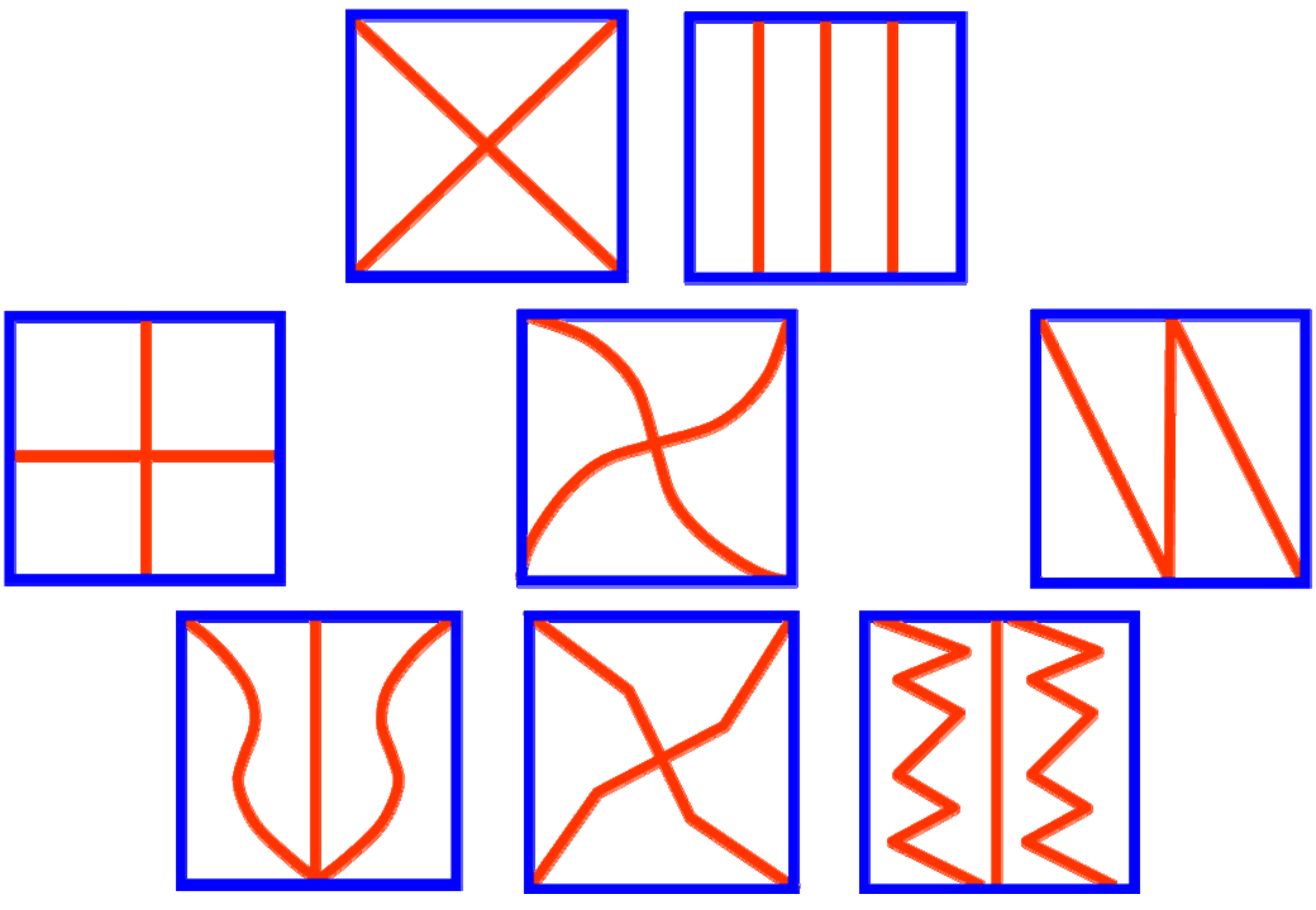
H) Exploring problems with infinite number of solutions

Divide a square into four identical pieces

This is a visual problem with an infinite number of solutions.



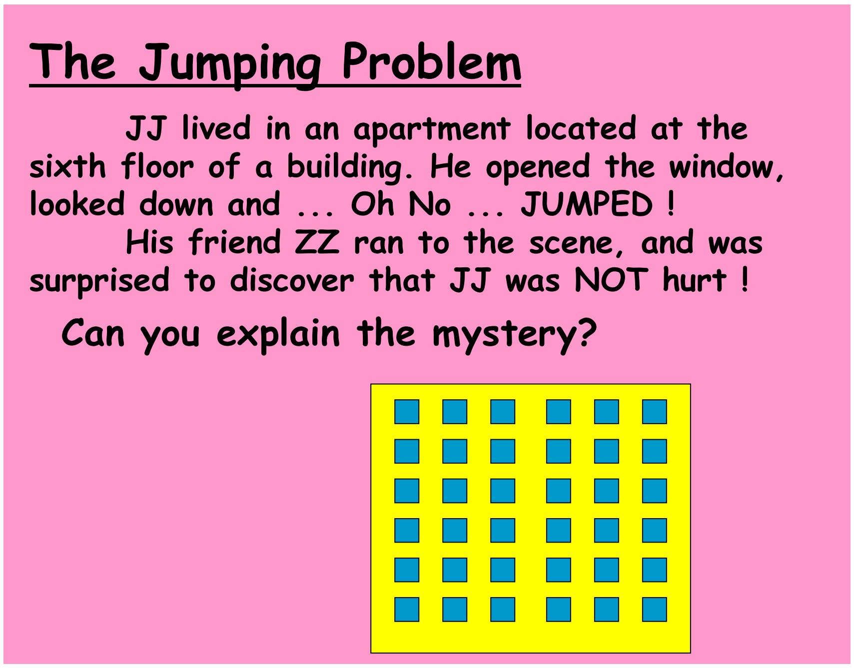
The following are some solutions:



I) Evaluation problems

The following two problems were used in evaluating class-divergent thinking

The Jumping Problem



Where are you?

Solutions to “The Jumping Problem”

JJ used a parachute
JJ landed in water
JJ landed in something soft
JJ was lying and didn’t really jump
JJ was bungee jumping
JJ jumped to a lower floor
JJ landed on a trampoline
JJ was a bird
JJ was a cat and landed safely on his feet
JJ was a stunt man
The window was a fire escape
JJ could fly
JJ had a jet pack or rocket
JJ tied a rope around himself and lowered himself to the ground
The building was underground so the sixth floor was at ground level
JJ was a superhero
JJ was already dead
JJ landed on a garbage truck
JJ landed in a net
JJ lived in a midget/dwarf apartment that was half the size as a normal building
JJ had a hang-glider
JJ used a ladder to climb down
JJ grabbed onto the water drain pipe
JJ jumped to helicopter
JJ jumped to next building
JJ was on the first floor and jumped
JJ caught his shoe lace on the side of the window
JJ jumped backwards
JJ landed on trees
JJ landed on ZZ
JJ was Spiderman and climbed down safely
There was a slide on the side of the building and JJ went down it
ZZ thought he saw JJ jump out of the window
JJ jumped in his own apartment
JJ jumped out of an indoor window
JJ landed on a window on a lower floor
JJ was immune to gravity
Someone caught JJ
Superman saved JJ
The building was on the moon

A fireman caught JJ
Each floor was only one foot high
JJ fell in the bushes
JJ is invincible
JJ was a set of twins
JJ believed he was a god and could not get hurt
JJ held onto the ledge
JJ jumped back inside
JJ jumped down onto the balcony below one by one
JJ jumped to a flying car
JJ jumped to helium balloon
JJ landed on someone
JJ was a cartoon character
JJ was using suction cups to walk down the side of the building
JJ was ZZ's imaginary friend
The building was built on a hill and JJ did not have far to fall
JJ had an experience to protect himself
JJ jumped into a pile of clothes
JJ jumped into his spaceship
JJ jumped to scuffling
JJ landed on an open truck full of features
JJ was hurt, but by the time ZZ got there he was OK
JJ was in a bubble and bounced back up
JJ's building is a dollhouse and JJ is a doll made of plastic
ZZ caught JJ
Angel saved JJ
Doctor on the ground fixed JJ immediately
It was a video game and JJ was an animated character
JJ fell onto a tent
JJ had a blast of air so strong it blew him back into his apartment
JJ had an anti-gravity belt
JJ had something to slow his descent
JJ had wings
JJ is a robot that cannot be hurt
JJ is in a movie and there is no such thing as a 6th floor apartment
JJ jumped onto large construction equipment
JJ landed in an ocean
JJ landed in an opened sewer
JJ landed in the arms of some cheerleaders
JJ landed on a carnival moonwalk
JJ landed on a flying carpet
JJ landed on a giant eagle
JJ landed on a hay stack
JJ landed on a pile of feathers
JJ landed on an open truck with full of hay

JJ opened the window in the corridor
JJ threw a dummy out of window
JJ used an umbrella to hover down
JJ was lucky
JJ was wearing a special suit that not let him get hurt
JJ wore spring shoes
JJ's building had burned to the ground so the 6th floor was the first
The building was sideways and all the floors were at ground level
The story is not complete
Wind helped him to fall slowly
Both JJ and ZZ were in virtual reality game
Both JJ and ZZ were insane people
Building had a lot of snow to 5th floor
Building had only one floor
Building was an underwater building
Building was flooded
Building was in space station
Building was not very high
Demon saved JJ
Ground was bounding rubber
His apartment was 6 stories high
It was a dream
It was not time for JJ to die
JJ can walk on air
JJ changed the charge of his body
JJ didn't hit the ground yet
JJ dove into a glass water cup
JJ drank a "Red-Bull" and it gave him "wings"
JJ fell just right
JJ floated down
JJ glided his way down
JJ grabbed an overhang
JJ had a blimp
JJ had a cushion
JJ had a mutant power
JJ had a new flying machine
JJ had a super watch
JJ had bouncing shoes
JJ had on anti-gravity boots
JJ had on special gear
JJ had strong bones
JJ had telekinetic power
JJ hovered in air
JJ is a ghost
JJ is a gymnast

JJ is an electronic airplane
JJ is like Rudolph the Red Nosed Reindeer
JJ jumped and grabbed 2nd floor escape
JJ jumped from a tied blanket to ground
JJ jumped in freight
JJ jumped in the room
JJ jumped into a hot air balloon
JJ jumped into the stairwell
JJ jumped on a windowsill
JJ jumped onto a crane
JJ jumped onto an elevator and rode it all the way down
JJ jumped sideways
JJ jumped to a cherry picker
JJ jumped to clothesline
JJ jumped to flagpole
JJ jumped to light-post
JJ jumped to log
JJ jumped to pipe and slide down
JJ landed in a manhole full of water
JJ landed in a pile of dirt
JJ landed in a river
JJ landed in snow
JJ landed in the arm of person
JJ landed on a baby carriage
JJ landed on a bed
JJ landed on a big bird
JJ landed on a big bird nest
JJ landed on a big pile of dust
JJ landed on a big umbrella
JJ landed on a car
JJ landed on a cart of pillows
JJ landed on a cloud
JJ landed on a convertible car
JJ landed on a group of cats
JJ landed on a group of chickens
JJ landed on a horse carriage
JJ landed on a jello pool
JJ landed on a laundry cart
JJ landed on a mayonnaise pool
JJ landed on a pile of boxes
JJ landed on a pile of dead people
JJ landed on a pile of wigs
JJ landed on a plane
JJ landed on a sandbox
JJ landed on a soft sofa

JJ landed on a soft spot
JJ landed on a some balloons
JJ landed on a tank of water
JJ landed on an open truck full of lettuce
JJ landed on Dumbo (flying elephant)
JJ landed on fish underwater
JJ landed on fuzzy fertilizer
JJ landed on giveaway
JJ landed on his feet
JJ landed on his friend
JJ landed on street dogs
JJ landed on tarps
JJ landed on the back of an elephant
JJ landed on the back of horse
JJ looked like he was alive
JJ opened the car window
JJ played a game
JJ slowed down time and was not hurt
JJ trained himself to jump out of 6th story buildings
JJ tried to clean the window
JJ was a Batman
JJ was a computer graphic
JJ was a doll
JJ was a flying squirrel
JJ was a kangaroo
JJ was a lizard
JJ was a monkey and swung down
JJ was a pain-free person
JJ was a stuffed animal that came to life and landed without getting hurt
JJ was an alien
JJ was blind
JJ was committing suicide
JJ was deep sea diving
JJ was disabled and could not jump
JJ was grabbed by alien
JJ was jumping in bed
JJ was on a pogo stick
JJ was six stories tall
JJ was stuck at the window
Someone else was disguised as JJ
The numbers on the floors were backwards: Biggest to Smallest
There were two JJs
ZZ called for help
ZZ did not think JJ was hurt
ZZ misidentified JJ

Solutions to the “Where are you?” Problem

Ask someone
Look at any street signs
Look for any familiar monuments or buildings
Buy a map
Look at the license plates of cars
Use GPS
Watch television
Listen to the peoples' accent around you
Call someone and have them look at your area code on caller ID
Look at the temperature / climate
Look for a police car that displays the name of the city
Look in a phonebook
Open a mailbox and look for the address
Look for the closest airport
Check the Internet to see where you are
Listen to the radio
Look at a newspaper
Look at the wildlife and vegetation
Look around for popular landmarks
Call the information center
Find the border or ocean and trace back how far you went
Look at the scenery / landscapes
See what type of clothing people are wearing
Explore until you know where you are and recognize things
Go up high in plane or helicopter to determine the type of land around you
Look at the stars
Look in an atlas
Use the Onstar system
Look at the telephone area code
Look at your plane ticket
Use a compass
Call the operator
Eat at a restaurant to see what food the city is known for
Get arrested and go to the county jail
Guess
Look for a "You are here" sign
Look for a highway or interstate
Look for a local taxi
Look for a post office
Look for a public building for a "City of ..." sign
Look for a satellite image of the area
Look for saltwater or freshwater (Water characteristics)

Send a letter to your home and check the postmark
Take the local public transportation to be able to recognize certain areas
Watch the weather channels
Cross into a new state and look at the sign
Find out the zip code
Find the city limit sign and read it
Look at local businesses
Look for a "Welcome to ..." sign
Look for a billboard
Look for the state flag
Retrace your steps
Compare the time difference from home
Look at how people drive
Look at the names of shopping centers and stores
Look at the sun positions
Look at the tourist information at a hotel
Look for business cards
Look for someone's ID
Look for state colleges
Look for tourism item for name of city
Observe local crops
Observe technology usage
Observe the other side of the earth
Observe the season and sun location
Start a fire and ask fire rescue
Use dog senses
Wake up from dream
Ask an alien
Buy a house an look at the papers
Buy something from a store that indicates the address
Call the FBI
Determine if the area is rural or a city
Do something crazy - then watch the news and find out where you did it
Find the nearest hospital
Fly to outer space and observe
Go to the next town
Locate the latitude and longitude of the area
Look for a national park
Look for a police or fire department
Look for a sport logo
Look for a train station
Look in a local magazine
Observe anthropology threats
Observe the local culture
See how many casinos there are

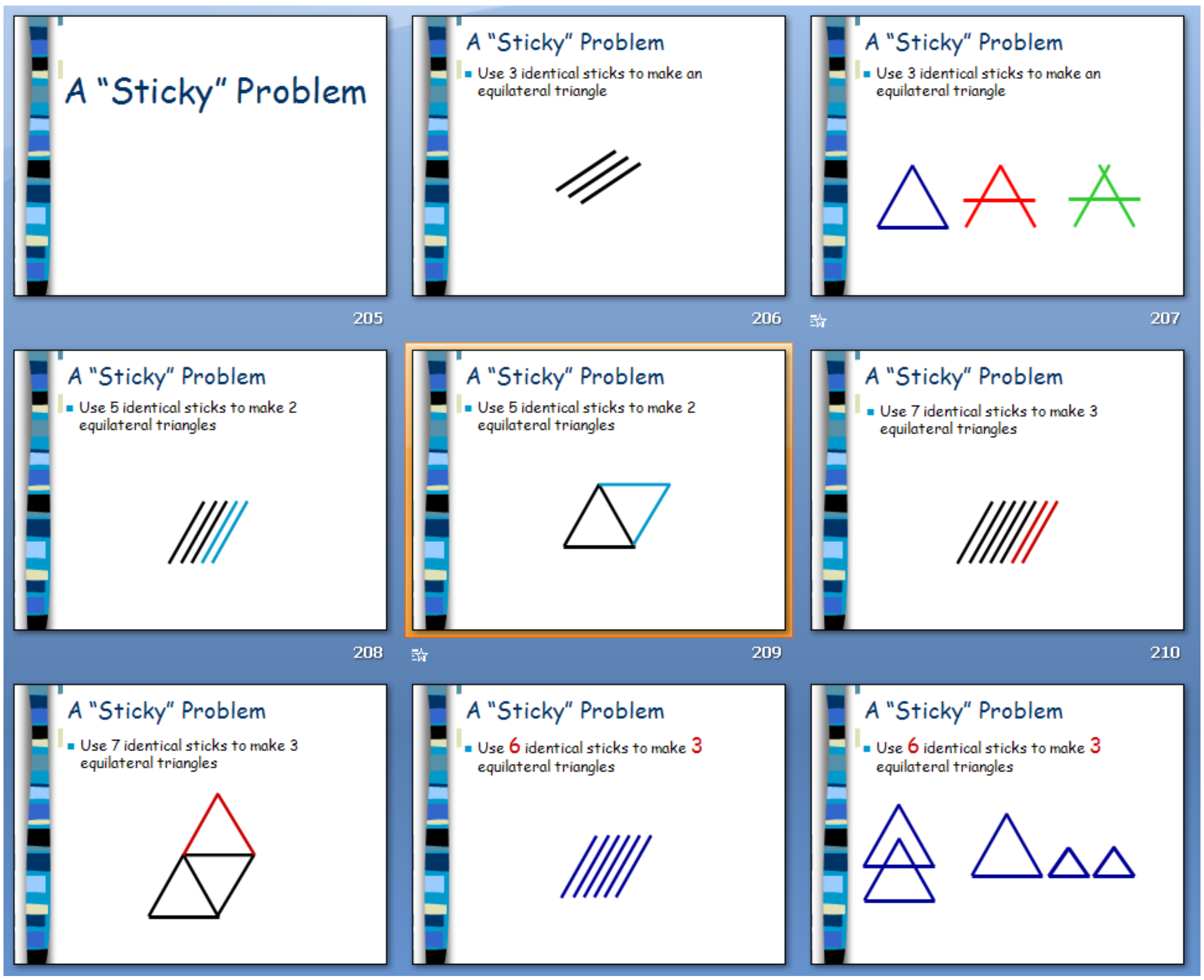
Use time machine to back a few days before
Wait for someone to look for you
Walk along the railroad track
Ask a bus driver
Ask someone in a local house
Ask someone on a CB radio
Ask someone on a HAM radio
Ask the children
Ask the friend who drove you
Attach a sign "Remind me where am I" on your back
Be smarter and realize by yourself
Become a secretary and look at addresses in files
Become a taxi driver
Become telepathic / psychic
Call 911
Call a movie theater and get directions
Compare the similarities and differences from your hometown
Determine the types of local industries
Find a congressman name to indicate district
Find a friend
Find out where you are not
Find the state capital
Flush the toilet
Follow clues like a scavenger hunt
Get a ticket and look at that
Go home
Go to a gathering
Go to concert and let singer tell you
Have Homeland Security pick you up - they will tell you during the interrogation
Hold up a sign "where am I?" until someone tells you
Hurt yourself and go the hospital
Identify from the unique characteristics
Just stay (Don’t worry about where you are)
Knock on someone's door and ask
Know where you are going
Let local people known that you don’t know where are you
Listen the local music
Listen to zip-code at store cash register
Look at the economic status of the city
Look at the power on the wall outlet
Look at the state lottery in gas station
Look at the type of people that are around you
Look at your feet
Look for a car dealership

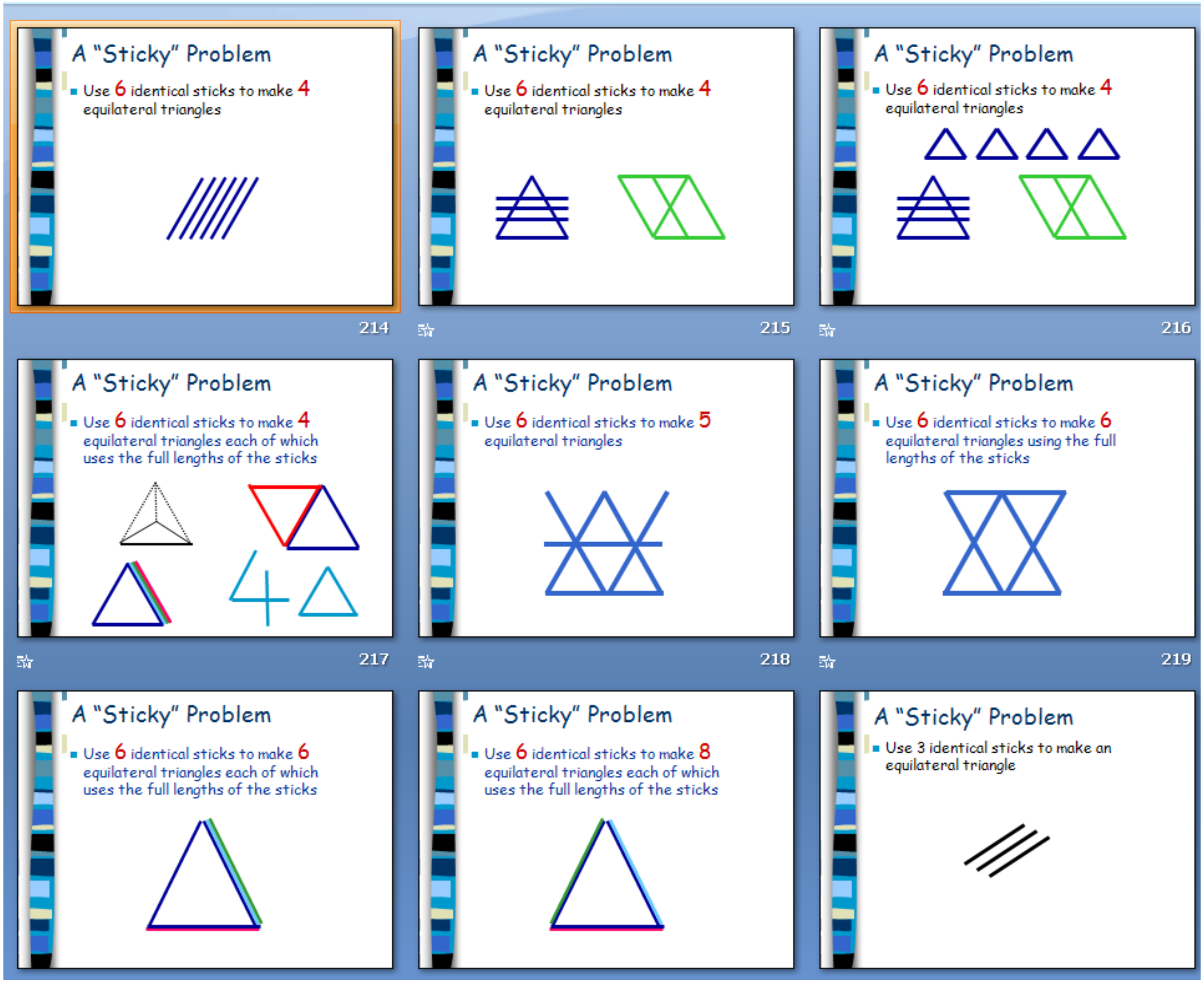
Look for a city park
Look for a sports stadium
Look for a state dog tag
Look for a subway station
Look for a tourist attraction
Look for farm animals
Look for local ads
Look for pay-phone booths
Look for the county that you are in
Look for the school district
Look for theme parks
Look in gift shops that might attract tourists
Look when someone fills out an application form
No Solution
Observe structures
Observe the distance between two cities
Observe the hobbies of local people
Observe the kinds of cars
Observe the local history
Observe the local law
Observe the population density
Observe the soil
Observe the wind
Pray
Report yourself as a missing person
Send a signal to ask
Shoot a laser and observe the angle of reflection
Take a train and look at the map inside
Trade something for someone to tell you where you are
Use a hot-air balloon
Use a pedometer and walk to Canada - find your location
Use a satellite phone
Walk around until you figure it out
Wave down a car
You know you are in the USA

J) Twists to ("very") well-known out-of-the-box problems

Use 6 popsicle sticks to make 4 equilateral triangles

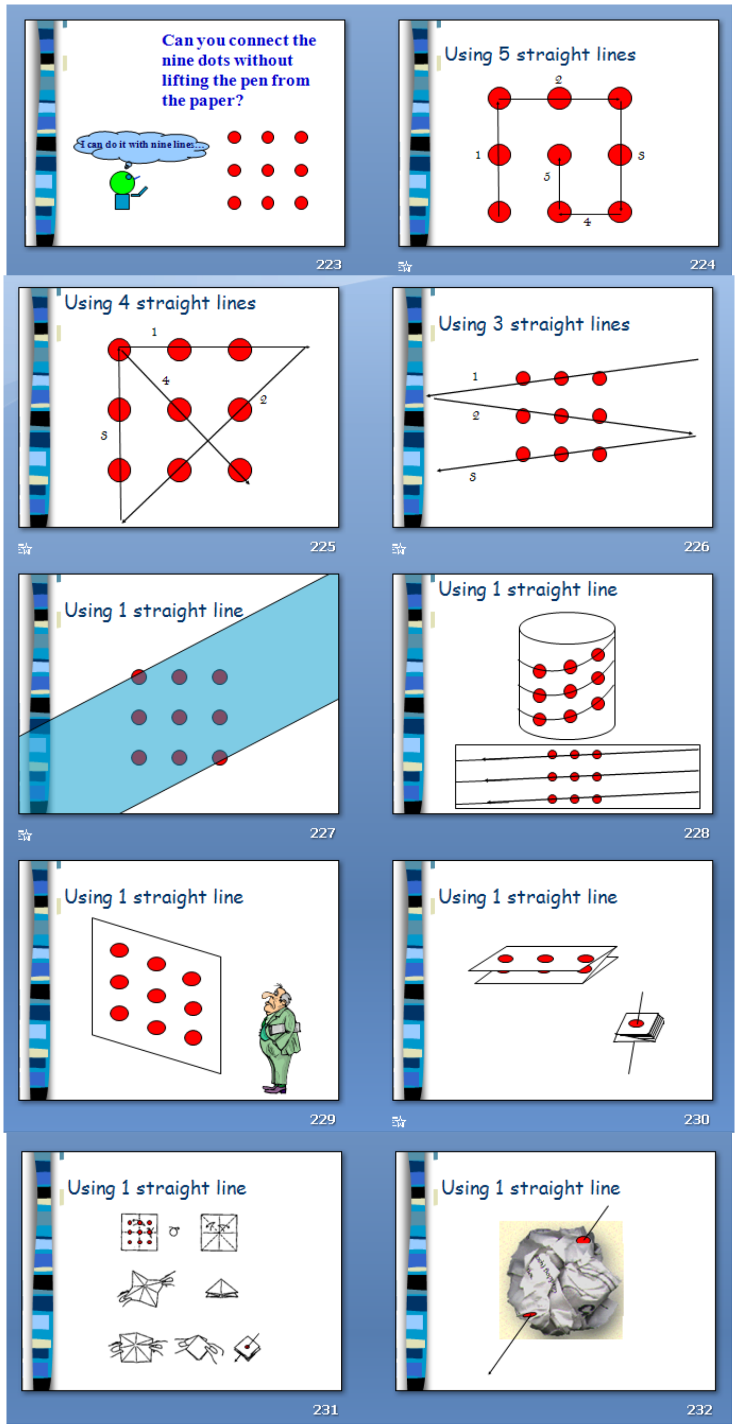
A well known problem: use 6 popsicle sticks to make 4 equilateral triangles. Students discover that by looking for a 3-D solution, the problem can be easily solved by constructing a pyramid. In this exercise we take the participants step by step to discover multiple solutions to different triangle-related problems (listed below). Apparently, even the original problem, i.e., "Use 6 popsicle sticks to make 4 equilateral triangles," has multiple solutions.




A "Sticky" Problem
Use 6 identical sticks to make 4 equilateral triangles
214
A "Sticky" Problem
Use 6 identical sticks to make 4 equilateral triangles
215
A "Sticky" Problem
Use 6 identical sticks to make 4 equilateral triangles
216
A "Sticky" Problem
Use 6 identical sticks to make 4 equilateral triangles each of which uses the full lengths of the sticks
217
A "Sticky" Problem
Use 6 identical sticks to make 5 equilateral triangles
218
A "Sticky" Problem
Use 6 identical sticks to make 6 equilateral triangles using the full lengths of the sticks
219
A "Sticky" Problem
Use 6 identical sticks to make 6 equilateral triangles each of which uses the full lengths of the sticks
A "Sticky" Problem
Use 6 identical sticks to make 8 equilateral triangles each of which uses the full lengths of the sticks
A "Sticky" Problem
Use 3 identical sticks to make an equilateral triangle

The nine dots problem

The well-known “nine dots” problem is being used to explore unexpected “out-of-the-box” multiple solutions to a problem. Students are asked to first connect the three rows of three dots in each row with <u>five</u> connected straight lines (very easy), then with four, then with three, and finally with one.



K) Exploring simple problems with unexpected solutions

Here is a problem that, at first glance, seems to be too trivial and with limited number of solutions. It turned out to be a great divergent-thinking exercise.

Problem: Make the following sentence complete and correct:
This sentence has ___________ letters.

Shown here are more than 230 different solutions, some of which can be generalized to become sets of finite multiple and even infinite number of solutions. In preparing the solutions I made every effort to avoid duplications. I hope there are none.
In parenthesis are brief explanations to some of the solutions.

MATHEMATICS–RELATED SOLUTIONS

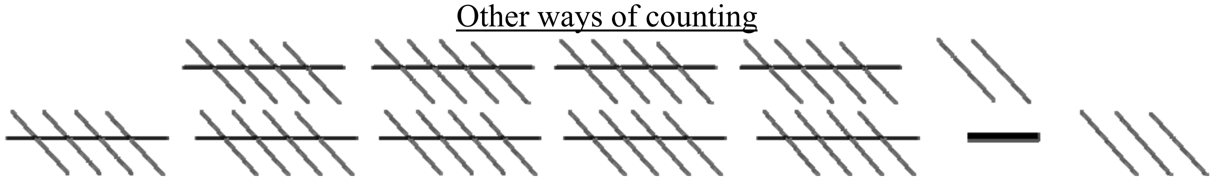
Numbers only

22
12 (counting the letters only in “This sentence”)
10 (counting the number of different letters)

Roman numbers only

XXV
XXVI
XXVII
XXVIII

Other ways of counting



Number operations

[(5!)/12]+2
IX . III (same as 9 x 3; “.” Stands for multiplication operator)
44/2 (or 66/3, etc.: infinite # of solutions)
20+2 (or 19+3, etc.: infinite number of solutions)
(Or other possibilities such as 18+4, 19+3, 20+1+1, 20+4-2, 4+8+3+7)
2.2 x 10
XXIX – II + 1 (using Roman letters for 29 and 2, and adding the number 1; the is total 28)
XXX – III + 1 (again, using Roman letters for 30 and 3 and adding the number 1; the is total 28)
F7 (using hexadecimal representation of 23)
10110 (using binary representation of 22)

Spelled-out numbers

thirty one
thirty three
thirty three plus nine
thirty plus seven
thirty three plus nine + two
only thirty seven
forty nine minus nine
one less than forty two
thirty eight (total)
thirty six (total)

Generalization of the above (adding a “phrase”/number to a given solution)

By adding , for example, “plus four” to the above solutions we get, for example:

thirty one plus 4
thirty three plus 4
thirty three plus nine + two plus 4
thirty plus seven plus 4

The above can also be generalized by adding, for example, “plus five + 3”:

thirty one plus five + 3
thirty three plus five + 3
Other related possibilities
thirty one + twelve/2
thirty three + twelve/2
thirty three plus nine + two + twelve/2
thirty plus seven + twelve/2
More options:
thirty one + dozen -7
thirty three + dozen -7
thirty three plus nine + two + dozen -7
thirty plus seven + dozen -7

Mix of number, operations and words

21 + eighteen/2
9 times 3
27 times 1
28 minus 1
twenty 8
twenty + 8
twenty – 8 (counting the letters in “This sentence” only)
twenty one -9 (again, counting the letters in “This sentence” only)
thirty + zero + 2
thirty plus zero + 6
thirty plus zero plus + 10
thirty + 2 + zero

forty - 11 + one
forty – 12 + two
forty – 13
forty plus one - 7
13 plus 13
27 times 1
29 minus 2
18 plus 8
17 plus 9
27 times 1
31 times 1 – 4 (similar to the above)
32 times 1 – 5 (similar to the above)
32 times 1 minus 0
33 times 1 minus 1 (This is similar to the above)
16 times 2 - 5
8 times 4 - 5
28+two+two+zero
30+zero+zero
34+zero+zero+zero
20 seven
2 plus 24 (or 24 plus 2)
9 vowel
a total of 30
2 plus 24
18-font size

Mix of numbers/words/phrases/descriptors
6 repetitive
12 different
27 thick
12 kinds of
exactly 29
, I think, 28 (the actual number is 28)
, I guess, 28 (the actual number is 28)
, in fact, 28 (the actual number is 28)
, practically, 33 (the actual number is 33)
, may be, 27 (the actual number is 27)
, perhaps, 29 (the actual number is 29)
, I believe, 30 (the actual number is 30)
, believe it or not, 36 (the actual number is 36)
twenty – 6 different (this means that the sentence has 14 different letters)
33 most amazing
(The specific number can be changed/generalized using a different number that is the sum of 22 and the number of letters in the words that follow the number, for example 31 fantastic, in this case it is 31=22+9)
22+4+7 most amazing (This is a version of the above solution)
21 LOWER CASE

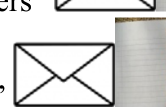
(or 22 LOWER CASe, 25 LOWER case, 30 lower case, etc.)
2 English capital letters
Two uppercase
12 unique
2 numbers and 32
3 digits and 31
10 underlined
2 dz
12 different
5 ‘t’
5 ‘e’
5 ‘s’
fourteen different
forty minus three
seven groups of

Approximations
approximately 34 (the actual number is 35)
approximately 36 (the actual number is 35)
almost XXXII (The roman number is 32. The actual number of letters is 33)
almost XXXIV (The Roman number is 34. The actual number of letters is 33)
About 18% t
almost 29 (the actual number is 28)
about 29 (the actual number is 27)

Inequalities
no words more than 8
(Obviously this can be extended to no words more than 9, etc.)
less than 45
(This of course can be extended to less than 46, etc.)
More than 18
(This can be extended to more than 17, etc.)

Different point of view

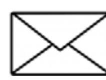
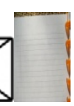
0 (when looking at different meaning of the word “letters” )

zero (when looking at different meaning of “letters” )

NON-MATHEMATICS–RELATED SOLUTIONS

Adjectives/descriptors
many
nice
lower case
unique

funny
an odd amount of
an even amount of
some
plenty of
enough
capital and lower case
A bunch of
too many
no envelops, only some
no envelops and no
boring
countable
repetitive
numerous different
invisible
bold, handwritten, typed
plenty of
easily readable
written
only
many quotes from my previous
no mistaken
a big hint in the
no upside down
mostly lower case
many quotes from my previous
no upside down
mostly lower case

no (referring to letters used in envelops )
English
Latin
Roman
Lots
no Chinese
no Egyptian
no Hebrew
no numbers, just
no meaning without
commonly used
no meaning without
finite number of
a "." and
spaces and
a "," and
no envelops and no letters

not only
no meaning without
no numbers, just
boring
countable
repetitive
words that are made by
words constructed from
created frustration by making me think about
increased my annoyance with
organized
limited
mostly lower case
more than I can count on my fingers
a secret number of
unpublished number of
fabulous
I do not care how many
I do not want to count how many
alphabetical
some alphabetical
many alphabetical
unexpected alphabetical
so much importance given by its
black
thick
repeating
awesome
too few
more than a few
too many
fair amount of
the best
the brightest
an abundant number of
not enough
no bold
consonant and vowel
enough
capital and lower case
beautiful
magical
energetic
too short
ordinary
some ordinary
ink

<u>ADDING logic operators such as “and” “or” to existing solutions</u>
tall and short and beautiful
punctuation and

<u>Adding a string to existing words</u>
-sles any writer, complicating it with (this will make the sentence: This sentence hassles any writer, complicating it with letters)
-sles me by requiring me to stare at (this will make the sentence: This sentence hassles me by requiring me to stare at letters)

<u>Extending the sentence</u>
so much importance given by its
a unique meaning represented by various
been extracted from a bunch of

<u>Doing nothing</u>
__________ (simply leave it blank)

<u>Other</u>
, I believe,

Problem
The year is 2006.
I was born 53 years ago in ‘53, and I am 35 years young. How come?
Provide solutions.

The following are some solutions:

-- 35 in Hex = 53 in Decimal, so 35 become 53
-- If you read 35 from right to left it becomes 53
-- 53 years old now, expecting to live another 35 years (in other words, I am counting backwards)
-- I feel/act like 35
-- sorry I am dyslectic
-- I took a trip in space ship near speed of light, and experienced time dilation (The Twin Paradox)
-- sorry I am lying
-- I was frozen for 18 years
-- I am a limited-edition factory-recertified automobile

The 7-11 problem

"A guy walks into a 7-11 store and selects four items to buy. The clerk at the counter informs the gentleman that the total cost of the four items is $7.11. He was completely surprised that the cost was the same as the name of the store. The clerk informed the man that he simply multiplied the cost of each item and arrived at the total. The customer calmly informed the clerk that the items should be added and not multiplied. The clerk then added the items together and informed the customer that the total was still exactly $7.11. What are the exact costs of each item?"

This problem appears in several sources (see References).

Traditional solutions:

The literature that deals with solutions to the well known 7-11 problem shows an exact solution: $1.20, $1.25, $1.50, and $3.16, and some approximate solutions e.g., $1.01, $1.15, $2.41, and $2.54.

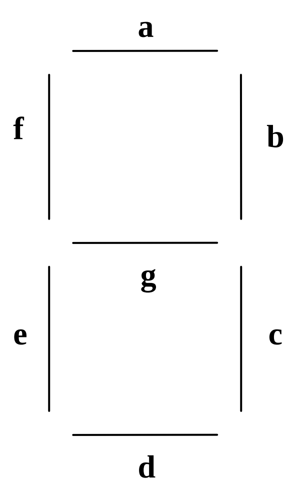
Non-traditional solutions:

When the 7-11 problem is introduced in my classes, students were asked to think in many other unexpected directions, leading to multiple solutions, based on:

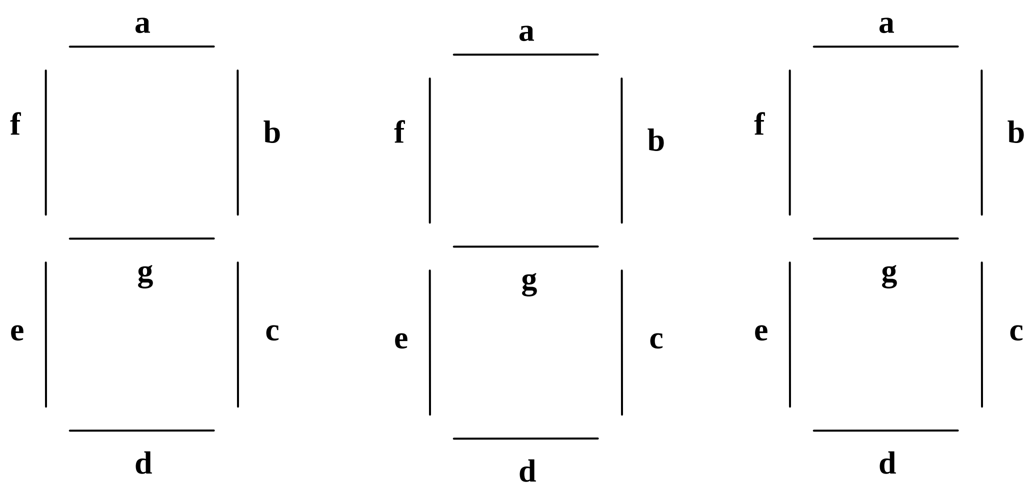
- 7-digit display in which some LEDs are burned or permanently on
- "Buy 1 get 1(or more) free" (or other deals)
- Including and excluding taxes
- Discounted prices
- Malfunctioning cashier calculator,
- etc.

New solutions to the 7-11 problem:

Let's represent each digit using a 7-segment LED display (see figure).



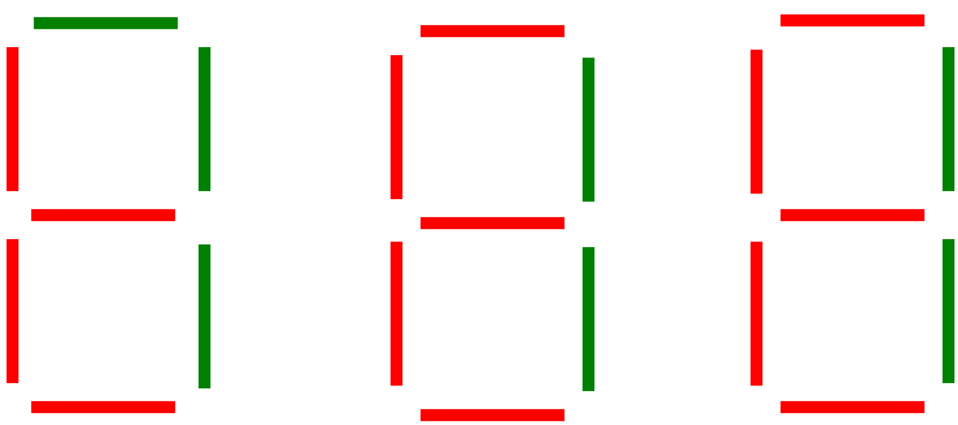
If the cash-register contains three such digits, where the left one represents dollars and the other two represent cents, then it looks like:



Obviously, if some of the LED segments are burnt, then some of the 0-9 digits will be represented incorrectly.

Let's assume that in the left 7-segment digit the only functional segments are a,b,and c, and that the segments d,e,f,and g are burnt.
Also assume that the two "cents digits" have only 2 functional segments, namely b and c and the rest are burnt.

In the following figure, the functional segments are shown in green and the burnt segments are shown in red.



Clearly, the current green LED's display "711".

In this specific configuration of segments, due to the burnt segments, the “7” of the dollar digit can represent represents the digits 3,7,8,9, or 0, and each of the “1”s of the right digits can represent 1,3,4,7,8,9, or 0.

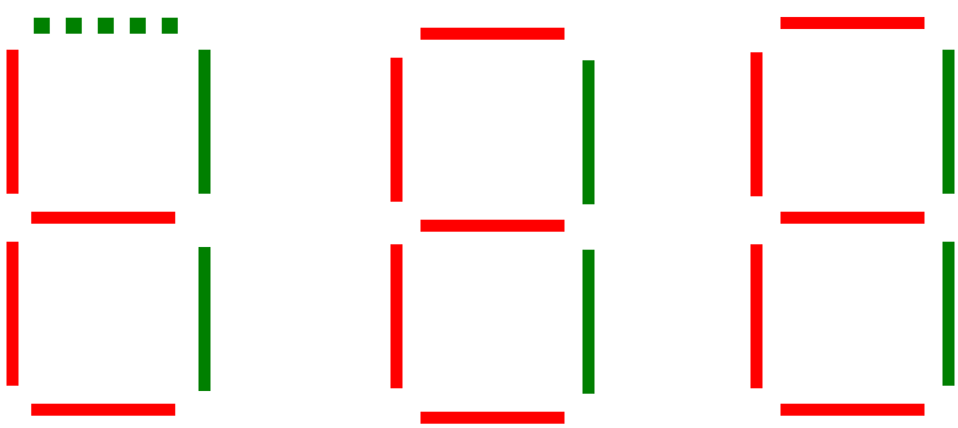
Now, back to the 7-11 problem.
If the price of an item is for example, 1.75 (the “$” sign was removed from the price), then:
1.75 x 4 =7.00.
This number will be represented by the above digit/segment configuration as “711.” This is due to the fact the some of the segments are burnt and the number 0 is mistakenly represented as the 1.
Also $1.75^4 = 9.38$ (the exact number is 9.37890625) and will also be represented by the above digit/segment configuration as “7.11” or if we ignore the decimal point it will simply become “711.” With the earlier described burnt segments, the number 9 will be mistakenly represented at 7, and the number 3 and 8 will be mistakenly represented as 1 and 1.

In the above example 1.75X4 (i.e., 7.00) is represented the same way as 1.75^4 (i.e., 9.38), as 7.11

Actually there are more solutions of this kind:
If the price of an item is 1.7775 (other numbers e.g., 1.777, 1.778, will be ok as well), then 1.7775 x 4 = 7.11 and $1.7775^4 = 9.98$ (about), both will be represented as “711”.

Here is another solution that is based on different assumptions:

There are other possibilities that are related to the 7-segment display:
If the “a” segment of the $ digit is “ALWAYS ON” (shown in green dotted line), and the rest functional/burnt segments are as before, then “7” in the left ($) digit can now mistakenly represent the numbers 1,3,4,7,8,9, and 0.



So if the price of an item is 1.00, then:
1.00 x 4 =4.00.
This number will be represented by the above digit/segment configuration as “711.”
Also $1.00^4 = 1.00$ and will also be represented as “711.”

Please note that we partially covered only one case where the price of all 4 items is the same. There are many other possibilities if we relax this assumption.

Note: There are other possibilities of burnt/functional/"ALWAYS ON" digits that can be explored.

For each of the two digits on the right, those that represent cents, without changing the original assumptions of the burnt segments, the two functional segments can be further manipulated to become

b=functional , c=ALWAYS-ON,

b=ALWAYS-ON, c=functional, or

b=ALWAYS-ON, c=ALWAYS-ON.

(The b=functional and c=functional possibility was discussed earlier.)

The "7" digit (the one on the left that represents $), without changing the original assumptions of the burnt segments, can be further manipulated to become

a=functional, b=functional , c=ALWAYS-ON,

a=functional, b=ALWAYS-ON, c=functional,

a=functional, b=ALWAYS-ON, c=ALWAYS-ON,

a=ALWAYS-ON, b=functional , c=ALWAYS-ON,

a=ALWAYS-ON, b=ALWAYS-ON, c=functional, or

a=ALWAYS-ON, b=ALWAYS-ON, c=ALWAYS-ON.

(The cases where b=functional and c=functional possibilities were discussed earlier.)

One obvious solution is the last one where all the non-burnt segments are ALWAYS-ON. It always results in "711" display regardless of the price.

Other solutions:

Let X represent the price of an item in $, and Y the tax in %, then:

1. $4\ X\ (1+ y/100) = 7.11$ and $X^4\ (1+ y/100) = 7.11$

And the solution to the set of equations is X=1.5874 Y=11.9755 (This means that the solution for item price is X= 1.5874 and the tax is about 11.9755%)

2. Buy one get one (or two) free:

$2\ X\ (1+ y/100) = 7.11$ and $X^2\ (1+ y/100) = 7.11$

In this case the solution is X=2 and Y=77.75

3. Buy one get three free:

$X\ (1+ y/100) = 7.11$ and $X\ (1+ y/100) = 7.11$

Since these are two identical equations with 2 unknowns, there are infinite number of solutions!

For example we can choose X to be 6.00, and the Y will become 18.5.

4. Assume that state taxes changed between one transaction to the other.

For example, from y% to z%.

Then,
$4 X (1+ y/100) = 7.11$ and $X^4 (1+ z/100) =7.11$
Again, infinite number of solutions!

5. Combine the 7-bar LED solution(s) with a possibility of the following additional computational error:

Instead of multiplying the 4 prices and then add taxes, the person calculated the price of each item including tax and them multiply the values.
$4 X (1+ y/100) = 7.11$ and $[X (1+ y/100)]^4 =7.11$

Referring to the first set of assumptions of the burnt segments that we described earlier
There is infinite number of solutions again!
Choose any combination that leads to $X (1+ y/100) =1.75$ to get a solution.

Example:
X= 1.40 and Y=25% results in total price of 1.75 (including tax) per item, and to "711" representation on the burnt/functional display as shown previously in the 7-bar LED solution.

6. This solution involves discounted price items

Buy the first item for a% discount, get the second item for b% discount, the third item for c% discount and the forth item for d% discount. This will result in infinite number of solutions. (If you add to it tax variable you get "many more" solutions.)

7. Combine the 7 bar LED solution(s) with solutions 1, 2 or 3.

More solutions:

8. It was July 11.

The machine displayed the date, 7-11, regardless of the item price.
(Or perhaps, the person at the cashier looked at the wrong display.)

9. The machine displayed the store name, 7-11, regardless of the item price.

(Or perhaps, the person at the cashier looked at the wrong display.)

10. Machine malfunctioned and always displayed 7-11.

11. In Roman numerals "711" is represented as DCCXI (=500+100+100+11).

From left to right it reads: IXCCD which is a strange way to write "289" (=500-100-100-9)
Then we get:
$4 X (1+ y/100) = 7.11$ (when looking at DCCXI) and $X^4 (1+ y/100) =2.89$ (when reading backwards, i.e., IXCCD)
The solution is
X=1.17588 and Y=51.16.

A twist to the above solution, based on an interesting observation:
7.11+2.89=10.00. Maybe the person at the cashier looked at the change from \$10 (\$2.89) instead of the value of the 4 items (\$7.11).

A **challenge for the reader**
(I received it from a student by e-mail; solution is not provided…)

The solution is unique. However there are many different ways to solve it

The problem:

Take the integers from 1 to 25 (inclusive) and arrange them in a straight line such that any two numbers next to each other will sum to either 2^i or 5^j where i and j are integers.

For example, pretend the following numbers are all on one line:
1 2 4 3 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

The numbers 12 and 13 are next to each other and they sum to 25 which is 5^2.

Also, 3 and 5 are next to each other and they sum to 8 which is 2^3.

However, 1+2 = 3 is not 2^i or 5^j for any integer i or j.

So, the above arrangement is not correct.

Students can submit the MIDDLE number.
(For the above incorrect example, this would be 13.)

Enjoy!

Results and Assessment

See reference 6 in Bibliography.

It should be noted that the results were collected and compiled by the teaching assistants of each class. In order to avoid any identification of participating students, the data given in this paper has been combined from all classes. However, the pattern of the overall data closely matches that of the individual classes. The number of solutions per student, the standard deviation, and the total number of different types of solutions were determined for each set of questions. It was sometimes difficult to define a “different” solution. Some seemed too similar and were combined, while others were left as different solutions. It is up to the individual to decide if jumping into a (presumably, stationary) pile of feathers is really different than jumping onto the back of a (presumably, mobile) truck filled with feathers. In this respect, the actual number of different solutions can not be known for sure, but the results show a clear pattern despite a few uncertainties.

There was some difficulty in determining the exact number of students participating. This is because in some cases, a student was absent or not registered in the course on one of the two days that the evaluation was conducted. This did not occur frequently, however, and has little impact on the overall trend shown by the results. According to our data, it is about six students out of about 130 participants.

Summary of Results

The detailed listing of all of the students’ responses can be found in the Appendix. It should be referred to in order to note the creativity and variety that was produced by the students. Tables 1 and 2 summarize the results for each question. “Before” refers to the evaluation given towards the beginning of the class and “after” refers to the evaluation given near the end of the course.

Where Are You?

Average Number of Students	Total Number of Different Solutions (Before)	Total Number of Different Solutions (After)	Average Number of Solutions Per Student (Before)	Standard Deviation (Before)	Average Number of Solutions Per Student (After)	Standard Deviation (After)
63	79	166	5.742	1.78	12.781	2.23

Table 1: Overview of results from “Where Are You?” question

The Jumping Problem

Average Number of Students	Total Number of Different Solutions (Before)	Total Number of Different Solutions (After)	Average Number of Solutions Per Student (Before)	Standard Deviation (Before)	Average Number of Solutions Per Student (After)	Standard Deviation (After)
64	94	220	4.969	1.34	12.25	1.63

Table 2: Overview of results from “The Jumping Problem” question

The following table depicts the number of students from each individual class that answered the given questions. The different numbers of students between "before" and "after" in a given class result from students being absent during one of the evaluation days.

Number of Students From Each Class

Class	Before	After	Question
1	10	10	Where Are You?
1	10	10	The Jumping Problem
2	7	7	Where Are You?
2	6	6	The Jumping Problem
3	11	11	Where Are You?
3	12	11	The Jumping Problem
4	9	9	Where Are You?
4	9	8	The Jumping Problem
5	14	15	Where Are You?
5	15	17	The Jumping Problem
6	11	12	Where Are You?
6	12	12	The Jumping Problem

Table 3: Breakdown of participants by question and class

Analysis of Results

The data shows a definite trend of an increase in ideation. On average, students generate more than twice as many solutions after completing the course. The quality of the answers was not considered in this study. It was the case that some students produced fewer, but more thoughtful or elaborate answers, while others had many short solutions. Each student interpreted the questions individually. One interesting note about the resulting solution is that one student described the location of the man to the buildings given as the image on the handout. This means that the use of only text, only pictures, or both to present the problem could alter the number and type of results produced. All of the students involved here, however, where given identical sheets on the same color paper as shown previously.

The increase in average number of solutions per student was 2.23 fold for the average number of solutions generated by the students for the "Where Are You?" problem. In addition, a total of 87 new solutions were generated by our classification. This means that the number of independent ideas doubled. There was a similar trend for "The Jumping Problem." On average, 2.47 times more solutions per student were generated between the first and second evaluation periods. Similarly to the first problem, the number of new solutions doubled with an increase of 126 ideas. Even with a somewhat large variance in deciding what a "different" solution is, there is clearly a meaningful increase in the number of different solutions produced as a whole.

Conclusion

This paper presents activities that are used by the author in several different problem-solving, creativity and innovation courses. They help the students to use new concepts in thinking and problem solving, to think differently, to use imagination, intuition and common sense, to appreciate others' points of view, and to have fun in the process. The exercises contribute to the development of a more innovative and creative classroom environment, and help a great deal in introducing students to problem-solving topics, such as "exploring more than one solution", "changing points of view", and appreciating diversity in thinking. As reported in previous papers by the author, at the end of the course students consistently generated many more solutions to given problems than at the beginning of the class (usually more than twice as many solutions). Assessing the benefits and drawbacks of each activity is a tough issue, and still needs to be worked on. In addition, for some students some of the activities may not be as fun as for others. In these special cases the instructor should be ready to intervene, help and share some hints to minimize the development of "mental blocks."

Assessment of some of the activities indicates a consistent and significant improvement in idea generation - a measure for innovative thinking. Results show an average increase in the number of ideas by a factor of nearly two and a half, produced by about 130 participants.

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http://soa.asee.org/paper/conference/paper-view.cfm?id=20677
8. Learning Systematic Problem Solving: Case Studies
http://soa.asee.org/paper/conference/paper-view.cfm?id=18147
9. Teaching Hands-on Inventive Problem Solving
http://soa.asee.org/paper/conference/paper-view.cfm?id=16476
10. The 7-11 problem:
http://jwilson.coe.uga.edu/emt725/Class/Lanier/Seven/Seven.html
http://jwilson.coe.uga.edu/emt725/7-11/711Hint.html
http://jwilson.coe.uga.edu/emt668/emat6680.folders/howard/7.11Problem/7.11.html
http://www.untruth.org/~josh/math/711.pdf